

Section 8.10 : Keplerian Elements for Approximate Positions of the Major Planets

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Lower accuracy formulae for planetary positions have a number of important applications when one doesn't need the full accuracy of an integrated ephemeris. They are often used in observation scheduling, telescope pointing, and prediction of certain phenomena as well as in the planning and design of spacecraft missions.

Approximate positions of the nine major planets may be found by using Keplerian formulae with their associated elements and rates. Such elements are not intended to represent any sort of mean; they are simply the result of being adjusted for a best fit. As such, it must be noted that the elements are not valid outside the given time-interval over which they were fit.

The elements are given below in Table 8.10.2 or in Tables 8.10.3 and 8.10.4, depending upon the time-interval over which they were fit and within which they are to be used.

Formulae for using them are given here.

8.10.1 Formulae for using the Keplerian elements

Keplerian elements given in the tables below are

- a_o, \dot{a} : semi-major axis [au, au/century]
- e_o, \dot{e} : eccentricity [radians, radians/century]
- I_o, \dot{I} : inclination [degrees, degrees/century]
- L_o, \dot{L} : mean longitude [degrees, degrees/century]
- $\varpi_o, \dot{\varpi}$: longitude of perihelion [degrees, degrees/century] ($\varpi = \omega + \Omega$)
- $\Omega_o, \dot{\Omega}$: longitude of the ascending node [degrees, degrees/century]

In order to obtain the coordinates of one of the planets at a given Julian Ephemeris Date, T_{eph} ,

1. Compute the value of each of that planet's six elements: $a = a_o + \dot{a}T$, etc., where T , the number of centuries past J2000.0, is $T = (T_{\text{eph}} - 2451545.0) / 36525$.
2. Compute the argument of perihelion, ω , and the mean anomaly, M :

$$\omega = \varpi - \Omega ; M = L - \varpi + bT^2 + c \cos(fT) + s \sin(fT) \quad (8-30)$$

where the last three terms must be added to M for Jupiter through Pluto when using the formulae for 3000 BC to 3000 AD.

3. Modulus the mean anomaly so that $-180^\circ \leq M \leq +180^\circ$ and then obtain the eccentric anomaly, E , from the solution of Kepler's equation (see below):

$$M = E - e^* \sin E, \quad (8-31)$$

where $e^* = 180/\pi e = 57.29578 e$.

4. Compute the planet's heliocentric coordinates in its orbital plane, \mathbf{r}' , with the x' -axis aligned from the focus to the perihelion:

$$x' = a(\cos E - e) \quad ; \quad y' = a\sqrt{1-e^2} \sin E \quad ; \quad z' = 0. \quad (8-32)$$

5. Compute the coordinates, \mathbf{r}_{ecl} , in the J2000 ecliptic plane, with the x-axis aligned toward the equinox:

$$\mathbf{r}_{ecl} = \mathcal{M}\mathbf{r}' \equiv \mathcal{R}_z(-\Omega)\mathcal{R}_x(-I)\mathcal{R}_z(-\omega)\mathbf{r}' \quad (8-33)$$

so that

$$\begin{aligned} x_{ecl} &= (\cos \omega \cos \Omega - \sin \omega \sin \Omega \cos I) x' + (-\sin \omega \cos \Omega - \cos \omega \sin \Omega \cos I) y' \\ y_{ecl} &= (\cos \omega \sin \Omega + \sin \omega \cos \Omega \cos I) x' + (-\sin \omega \sin \Omega + \cos \omega \cos \Omega \cos I) y' \\ z_{ecl} &= (\sin \omega \sin I) x' + (\cos \omega \sin I) y' \end{aligned} \quad (8-34)$$

6. If desired, obtain the equatorial coordinates in the “ICRF”, or “J2000 frame”, \mathbf{r}_{eq} :

$$\begin{aligned} x_{eq} &= x_{ecl} \\ y_{eq} &= + \cos \varepsilon y_{ecl} - \sin \varepsilon z_{ecl} \\ z_{eq} &= + \sin \varepsilon y_{ecl} + \cos \varepsilon z_{ecl} \end{aligned} \quad (8-35)$$

where the obliquity at J2000 is $\varepsilon = 23^\circ 43.928$.

8.10.2 Solution of Kepler's Equation, $M = E - e^* \sin E$

Given the mean anomaly, M , and the eccentricity, e^* , both in degrees, start with

$$E_0 = M - e^* \sin M \quad (8-36)$$

and iterate the following three equations, with $n = 0, 1, 2, \dots$, until $|\Delta E| \leq tol$:

$$\Delta M = M - E_n - e^* \sin E_n \quad ; \quad \Delta E = \Delta M / (1 - e^* \cos E_n) \quad ; \quad E_{n+1} = E_n + \Delta E. \quad (8-37)$$

For the approximate formulae in this present context, $tol = 10^{-6} degrees$ is sufficient.

8.10.3 Approximate Accuracies of the Keplerian Formulae

Table 8.10.1 gives the accuracies that one can expect from the Keplerian formulation given in this section

Table 8.10.1

Approximate errors, in right ascension, α , declination, δ , and in distance, ρ , from the Keplerian formulation of the present section.

	1800 – 2050			3000 BC to 3000 AD		
	α ["]	δ ["]	ρ [1000km]	α ["]	δ ["]	ρ [1000km]
Mercury	15	1	1	20	15	1
Venus	20	1	4	40	30	8
EM Bary	20	8	6	40	15	15
Mars	40	2	25	100	40	30
Jupiter	400	10	600	600	100	1000

Neptune	30.06952752	0.00895439	1.77005520	304.22289287	46.68158724	131.78635853
	0.00006447	0.00000818	0.00022400	218.46515314	0.01009938	-0.00606302
Pluto	39.48686035	0.24885238	17.14104260	238.96535011	224.09702598	110.30167986
	0.00449751	0.00006016	0.00000501	145.18042903	-0.00968827	-0.00809981

Table 8.10.4

Additional terms which must be added to the computation of M for Jupiter through Pluto, 3000 BC to 3000 AD, as described above.

	<i>b</i>	<i>c</i>	<i>s</i>	<i>f</i>
Jupiter	-0.00012452	0.06064060	-0.35635438	38.35125000
Saturn	0.00025899	-0.13434469	0.87320147	38.35125000
Uranus	0.00058331	-0.97731848	0.17689245	7.67025000
Neptune	-0.00041348	0.68346318	-0.10162547	7.67025000
Pluto	-0.01262724			